TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 6 2016

Calculator Free

Mathematical induction and complex numbers

STUDENT'S NAME _____

DATE: **TIME:** 50 minutes **MARKS**: 51

INSTRUCTIONS:

Standard Items: Pens, pencils, ruler, eraser.
Special Items: Formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

If $(a + bi)^2 = 3 + 4i$, where a and b are real numbers, determine the values of a and b.

2. (3 marks)

Determine the quadratic equation whose roots are $1 + 5i$ and $1 - 5i$.

3. (4 marks)

One root of the equation $z^2 + az + b = 0$, where a and b are real constants, is $2 + 3i$.
Determine the values of a and b.

4. (6 marks)

The complex number z satisfies $\frac{z}{z+2} = 2 - i$. Determine the real and imaginary parts of z. (Hint: let $z = a + bi$) .

5. (9 marks)

Simplify the following complex expressions leaving your answer in the form $a + bi$

(a) $2 - i - (-3 + 2i)$ [1]

(b) $(3 - 2i)(-2 + 5i)$ [2]

(c) $\dfrac{-3 - i}{2 + 3i}$ [4]

(d) $\dfrac{i}{-i^3}$ [2]

6. (5 marks)

Using the principle of mathematical induction prove

$2^0 + 2^1 + 2^2 + ... + ... 2^n = 2^{n+1} - 1$ for $n \geq 0$

7. (6 marks)

Using the principle of mathematical induction prove that $9^n - 2^n$ is divisible by seven for $n \in \mathbb{Z}^+$

8. (7 marks)

Prove $\begin{bmatrix} -2 & 9 \\ -1 & 4 \end{bmatrix}^n = \begin{bmatrix} -3n+1 & 9n \\ -n & 3n+1 \end{bmatrix}$ for $n \geq 1$ using mathematical induction

9. (6 marks)

Prove that $\cos x + \cos 3x + \cos 5x + ... + \cos [(2n-1)x] = \dfrac{\sin 2nx}{2\sin x}$ for $n \in \mathbb{Z}^+$